Testing volatility autocorrelation in the constant elasticity of variance model

Gianna Figà-Talamanca
Dipartimento di Economia, Finanza e Statistica
Università di Perugia

June 10\textsuperscript{th}, 2008

Since its first introduction many suggestions have been proposed for the generalization of Black and Scholes option pricing model (Black and Scholes, 1973). One of the most widely spread approaches is allowing for random volatility of the underlying stock price process as in the seminal papers by Hull and White (1987), Scott (1987), Wiggins (1987), Stein and Stein (1991) and Heston (1993). A complete list of references on stochastic volatility is beyond the scope of this paper.

Stochastic volatility models (SV models hereafter) account for many empirical facts in the stock and in the derivative markets, such as the leptokurtosis of financial log-returns and the so called smile curve of options’ implied volatility when plotted against the options strike price (see, among others, Cont, 2001).

The estimation of stochastic volatility (SV) models is still a challenging issue; recently, Ben-Hamida and Cont (2001), Ewald and Zhang (2004) and others have suggested different techniques in order to estimate model parameters in SV models by fitting theoretical option prices to market ones (model calibration). This approach has brought a renewed interest for the Heston and the GARCH diffusion models; in fact, a quasi-closed formula is available for European option prices in the former case (see Heston, 1993) and Barone Adesi et al. (2005) derive an approximated analytical option pricing formula for the latter. The calibration to market option prices is, thus, rather straightforward in both settings.

However, from a model risk minimization perspective (see Cont, 2006), before estimating a model, one should perform a preliminary analysis to test whether the model reflects some properties of observed data (e.g. moments, serial dependence etc.).

This paper focuses on the Constant Elasticity of Variance stochastic volatility (CEV SV) model which includes both the Heston model (Heston, 1993) and the GARCH diffusion introduced in Nelson (1990).

By taking advantage of the results in Genon-Catalot et al. (2000) , it is proved that, if the data generating process (DGP hereafter) of a stock price is a CEV
SV process in continuous time, then the sample autocovariance and autocorrelation functions of the stock squared returns, suitably scaled, are consistent and asymptotically normal estimates of the theoretical autocovariance and autocorrelation functions of the mean variance respectively; the asymptotic variances of these estimators are derived explicitly and approximate confidence bands are obtained for the corresponding theoretical values.

Moreover, a method to validate CEV SV model is suggested: given a finite set of discrete observations for a stock/index price the CEV SV model is consistent with observed data if the theoretical autocovariance/autocorrelation of the mean variance process lies in the confidence bands estimated from the sample.

For the illustration of the procedure, the sample autocovariance and the sample autocorrelation functions of the squared returns for 6 major stock Indexes (DJIA, S&P500, FTSE, DAX, CAC40 and SMI) are computed in order to obtain the estimated confidence bands for their theoretical counterpart. The evidence on observed data is that the CEV SV model gives a good description of the autocovariance/autocorrelation structure.

References


